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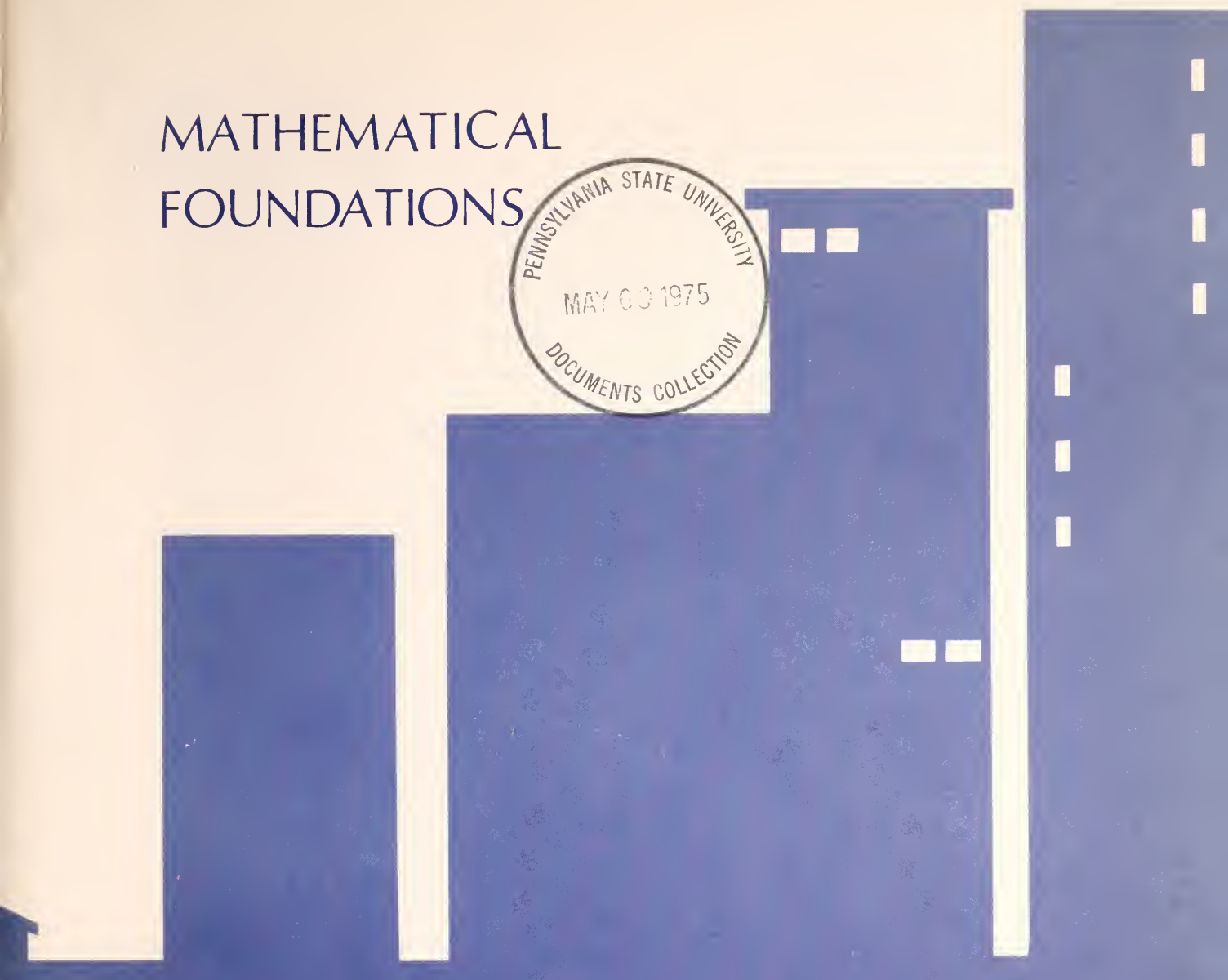

CITY GAMES

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CITY GAMES

MATHEMATICAL FOUNDATIONS

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National Bureau of Standards
Washington, D. C. 20234

November 1973

Final Report

Sponsored by
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

U. S. Department of Commerce, Frederick B. Dent, Secretary

National Bureau of Standards, Richard W. Roberts, Director

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Acknowledgement

We wish to acknowledge the significant contributions made by Mr. S. Guiland of Johns Hopkins University for his tireless effort in proof-reading the draft of this text. Also the efforts of Mrs. R. Ciufolo, Mrs. L. Groves and Miss C. Naas for their typing and editing of this manuscript is appreciated.

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I. INTRODUCTION

The CITY models are operational simulation games in which the participants make economic, government and social decisions affecting a hypothetical metropolitan area. Through the use of a computer, the simulated urban system responds to the participant's decisions as any real city would. Each player is assigned to a team which shares an economic, government or social role. The interrelated decisions made by teams will guide the way the simulated city changes in composition and size.

The simulation approach to cities offers the players an opportunity not only to make decisions but to implement them as well. They receive a feedback from their actions and see the effects from other forces that are constantly at work altering the effectiveness of the player's decisions. Players therefore have a learning experience on how to deal with a changing environment. The round-by-round play gives the players the necessary experience in selecting the type of analysis to move them towards their objectives while the allocation of their time and Game resources is a critical determinant of the success they hope to achieve. As the Game progresses, players learn to increase their involvement in the management of the environment while at the same time learning about the relationships between business and society.

The CITY model is really a series of Games. CITY I is the smallest of the Games consisting of the Economic and Government decision makers and operates on an IBM 1131 computer. It is primarily designed for land use and business decisions with the Government Departments acting more in the role of provider of services rather than regulatory agents. TELECITY or CITY II is CITY I with a partially operative social sector and operating on a Univac 1108 Exec 8 from a terminal. CITY III and CITY IV contain the Economic and Government decision makers and further expand CITY I by including the Social decision makers. The primary difference between CITY III and CITY IV is that CITY III is run on a Univac 1108 computer and CITY IV is run on an IBM 360/370 operating system. CITY IV was further expanded to include the water supply system known as the "water module." When the "water module" is used with CITY IV, the model is then known as

the River Basin Model.

To simplify the description of the theory of these modules, this manual is written for the four module game. The equations for the other games can easily be obtained by eliminating the descriptors that do not apply to the particular game of interest. The theoretical structure of all models are consistant and relevant only within the range from a central city area to a regional configuration. The description of the module components and equations are meant to show the scope of decisions including those by the users and by the programs and algorithms of the computer model.

All models have associated with them Players', Directors' and Operators' manuals which should be used in conjunction with this manual. The numerous details and particulars of each Game are described in the other manuals whereas this manual describes the theoretical base for all Games in the series.

To orientate the reader to this manual, it must be emphasized that all models are based on a grid matrix to locate and tabulate the larger number of interrelationships and to locate specific parcels of land or highways. Within each square or grid, numerous restrictions and/or data are imposed by each module. If a player and/or computer change is proposed for a particular location in the matrix, all conditions must be met or the proposal will not be accepted by the computer. The computer therefore acts as a large accounting system with minor simulation roles played in external relationships to the theoretical city such as the business cycle or export sales. The players serve as a decision making body which tries to optimize their decisions for the purpose of achieving individual objectives. This optimizing process might maximize individual goals but when placed in competition for a finite number of available resources, it might well be suboptimal for the entire city. It is within this framework that learning occurs. The numerous interrelationships must be negotiated before any system proposal can be obtained to improve the city as a whole. The computer will consider all proposed changes and allow only those changes which meet all constraints for a particular grid. In general, this process is expressed in mathematical form as:

$$D = \sum_{i=1}^{n} dm_i + \sum_{i=1}^{n} \sum_{j=1}^{m} dh_{ij} S_{ij}$$

where

S_{ij} = one-zero variable
- 1 - the decision by player j at location i is valid
- 0 - invalid decision

D = Total number of actual decisions made by all individuals at all grid locations accepted by the computer as valid

dm_i = Decisions made by algorithms designed into the model for all grid locations i=1,2...n

dh_{ij} = Decisions made by the players summed across all players at all locations.

The total number of possible decisions, say TD, will always be TD $\geq$ D since conflicting proposals can occur for the same grid or land parcel. For instance, the social players might propose a park for a particular land parcel and the economic sector might propose a building on the same site. The computer would then reject the two proposals as conflicting and TD > D. One of the objects of the game is to make TD $\rightarrow$ D which implies negotiation and well informed players.

As mentioned earlier, the decision process is constrained by resources available to the players and to the computer. All games view resources in two general categories, fixed and liquid. The total resources (TR) of a particular system can be viewed as the sum of all fixed and liquid resources in the system. The fixed resources (FR) include water, land, people, buildings, roads, networks (utilities, transportation, communication, etc.), capital equipment and the related maintenance items necessary to keep in operation. In contrast, liquid resources (LR) are those items which can easily be changed in the short run such as taxes, wages, sales, social attitudes and the like. The total resources of a system are merely:

$$TR = \sum_{p=1}^{n} FR_p + \sum_{i=1}^{m} LR_i$$

where

p = parcels or grid locations and

i = entities of LR

Since player and computer decisions are always constrained by the total resources available, the uniqueness of the model lies in the method for relating a wide variety of specific resources to each other in a highly interactive fashion. To capture the spirit of these interactions, all mathematical descriptions will take the form of a definition of the resources to be allocated or the receptors to handle the resource, and an objective function describing the rationale for the allocation. The receptors are usually balanced by a matching function using supply and demand relations. For example, if (TR) represents the total resources as described earlier, realistically every resource available would not be changed in a single round of the game. This is particularly true for the change in FR. Land owners may not want to alter all of the property holdings or the government might be satisfied with prior decisions and not want extensive changes to the "status quo." To express this idea of partial resource allocation (PR), as a mathematical function we write:

$$PR = \alpha CP + \beta CG + \gamma T + \varepsilon P$$

where

PR is the partial resource allocation

CP is private liquid or semi-liquid capital

CG is public liquid or semi-liquid capital

T is available time

P is the sum of all public and private influence

α, β, γ, ε are components which equate all of the resources to a least common denominator

The objective function for PR is conceived of as balancing the possible resource allocations such that there is a tendency to maximize the total resource base of the system. This partial resource objective function (PRO) is expressed as:

$$PRO \simeq \phi_C + \sum_{i=1}^{n} PR_i$$

where

PRO is the Partial Resource Objective Function

ϕ_C is added to represent those decisions which are made on a seemingly random basis

PR_i is the specific partial resource i

Since the player input equations are normally optimizing or satisfying a rate of return or benefit ratio from the allocation of a finite set of resources, it follows that the player objective function is merely:

$$\max \sum_{i=1}^{n} PR_i \quad \text{from above.}$$

This objective function represents all players by summing over all PR_is. Each individual PR_i is taken to be of the form:

$$PR_i = \frac{\hat{B}_i - C_i}{V_i}$$

where

PR_i is the partial resource allocation of specific partial resource i

$\hat{B}_i$ is the estimated benefit from project i in the form of dollars earned, service provided, expenditures saved, etc.

C_i is the known expenditures such as current expenditures for services, etc.

V_i is the total value of the project

In all cases there must be some procedure established for choosing among the many possible projects and this choice may be either the optimizing or satisfying assumptions. Associated with each PR_i is a project cost C_i. There is a limitation of the values of PR_i that will be acceptable and a limitation that the sum of the costs (C_i) will not exceed the available resources as measured in money, time or influence. Note that PR_i is indeed a rate of return.

The computer equations are designed to balance or allocate supply and demand. The general computer demand equation is:

$$DC = \theta_1(X_{x_1,x_2\text{------}},Y_{y_1,y_2\text{------}})$$

where

DC = demand (computer)

X is some set of local system demanders such as population groups, businesses, etc.

x_n are descriptors of the demanders such as class for population units and land use type for business, etc.

Y is some set of local system decisions such as time allocations for population units, operation level for business, etc.

y_n are descriptors of the decisions such as parcel location for population groups, maintenance levels for businesses, etc.

θ_1 is a quantifier or scale for the units of demand.

In the same manner the computer model equation for supply is written:

$$SC = \theta_2(W_{w_1,w_2\text{----}},Z_{z_1,z_2\text{----}})$$

where

SC = supply (computer)

W is some set of local system suppliers such as housing units, employers, stores, etc.

w_n are the fixed descriptors of these suppliers in the form of housing types, workers per level, design capacity, etc.

Z is some set of local system decisions such as rents and quality of housing, salary level for employees, prices at stores, etc.

z_n are the descriptors of the decisions such as locations for residences, businesses and stores, etc.

θ_2 is a quantifier or scale for the units of supply.

Throughout this manual, these general form equations will be used to describe the various game parts. General equations are given in all cases and where necessary specific equations are detailed to clarify the general equations.

The reader of this manual is advised to use a Players, Directors and Operators manual to clarify details which may not be fully explained in this manual. If the program is used as a reference, slight differences will occur in the variable names. Some of the symbols used in the main program are also used in subroutines with different meanings. To protect the continuity of this manual, duplicate symbols found in the program have been changed in this manual so that one definition is assigned to each variable. In all cases the program symbols are used.

II. PRIVATE CAPITAL ALLOCATION

In the general equation of partial resource allocation (PR), the term CP was introduced and defined as private liquid or semi-liquid capital. It was further defined that all capital resources would not be used during a particular game play which in turn led to the partial resource allocation concept. Assuming this concept to be realistic, it follows that any consideration of the total money stock available in a round for allocation may be defined as:

$$MS = \sum_{i=1}^{n} (S_i + dI_i + (aB - dC)_i)$$

where

MS	is the total money stock available in the system for allocation in a given round
n	is the total number of players who have money
S_i	is the available savings on private liquid capital for player i
dI_i	is the present value of the capital investment of player i
$(aB-dC)_i$	is the net amount of investment capital available after all costs of borrowing are taken into account for player i
aB	is the amount of money borrowed
dC	is the present value of the cost of borrowing

A. Choice of Investment

When the cost of making an investment becomes a larger and larger part of the money supply available, an investor is more apt to rank the net rates of return for his investment and choose those that will give him a greater rate of return. In terms of probability one might say that the probability that the net rate of return chosen is the one which gives the maximum rate of return tends towards one as the available investment money

supply diminishes to zero or the cost of the specific investment chosen increases to the money supply. Expressed mathematically:

$$p(\bar{N} = N_{max} | MS \to 0; K_i \to MS) \simeq 1$$

where

- p is the probability function
- $\bar{N}$ is the net rate of return on the chosen investments
- N_{max} is the maximum rate of return
- MS is the money supply
- K_i is the total cost of player i's series of investments composed of the actual cost of the investment and the cost of obtaining investment information

The probability statement can be further defined if one lets $N_i = r_i - C_i$ where N_i is the net rate of return for i's investment series, r_i is the estimated rate of return from i's investment series, and C_i is the cost of using money on i's investment series.

It then follows that $N_{max} \geq N_i$ and if μ is the error bound in estimating the net rate of return, then $N_i - \mu \geq 0$.

Since all liquid capital is assumed to at least earn a rate of return equal to the current rate of interest, it is assumed that the rational decision maker, i, would tend to order his investment series so that the first capital used would be borrowed as long as

$$(aB - dC)_i \leq r_i$$

That is, the net amount of investment capital available after all costs of borrowing are taken into consideration must be less than or equal to the estimated rate of return in dollars. When it becomes no longer profitable to borrow, i.e. $(aB - dC)_i > r_i$, the saving S_i and then the present value of invested capital dI_i would be considered. The invested

capital is only considered when the series of investments from the previous round (t - 1) yields a greater present value than that of a rearrangement of the investment series in the present round, which is expressed as

$$dI_{i_{(t-1)}} < dI_{i_t}$$

B. Types of Investments

Investors might wish to invest for long range potential or to make quick profits for a single round. In each of these cases, different objectives and methods for analyzing the investments must be considered.

In the long run, the investor is primarily concerned with the average rate of return to be expected for each investment. As long as the average expected value of the investment choice is greater than the cost, investment will take place. Decisions in this mode range from ones which consider any investment that is profitable on the average to more careful choices which can be ranked from the most to least profitable.

To clarify the above process and relate the process to the computer, one must first visualize the various players and then investment process independent of the computer. An individual player has limited resources to invest and will not always invest all resources in any given round. For instance, a player might be saving for a future investment or be content with his past investments and not want to invest at all during a current round.

Consider each player who has investment potential on a single parcel. A player might be from the Economics, Government or Social sectors and the investment potential will be biased toward the individual player's point of view. When individual investment is viewed this way, one can reason that the player would consider his investment associated with some probability of obtaining the benefits as the player visualizes and some probability that either no investment is made on the parcel or that a proposed investment will be invalid for violations of zoning or the

like. Viewed this way, one can construct a player investment equation to express the expected value of the net rate of return on all investments for a specific parcel. This can be expressed as:

$$E\tilde{N} \simeq \sum_{i=1}^{n} \frac{\hat{B}_i - DK_i}{V_i}$$

where

$E\tilde{N}$ is the expected value of the net rate of return on all investments possible on a particular parcel

n is the number of potential investors for the parcel

$\hat{B}_i$ is the estimated benefit of the ith player to invest on the parcel

dK_i is the cost of the investment distributed over time

V_i is the total value of the investment series to player i

In this context, $E\tilde{N}$ becomes a random variable in terms of the investment potential for a single parcel built upon an individual's probability assessment summed over all players who have potential to invest in the particular parcel. This is another way of expressing the thought process of individual players before submitting their decisions to the computer. One can further theorize that there is a maximum expectation possible on a given parcel ($E\tilde{N}_{max}$) and that $E\tilde{N}_{max} \geq 0$ for all parcels. The probability restrictions imposed by the money stock given earlier as:

$$P(\tilde{N} = N_{max} | MS \to 0; K_i \to MS) \simeq 1$$

limits the value of $E\tilde{N}_{max}$ to

$$\lim_{\tilde{N} \to 0} E\tilde{N}_{max} = E\tilde{N} \text{ providing}$$

$$V \geq dk \; \forall \; K_i$$

is based on the concept of marginal analysis where the greatest rate of return is sought for each land use at a specific location. Again, the decision possibilities range from situations where any positive rate of return is acceptable to when choices are carefully selected by ranked maximum rates of returns. Expressed mathematically, this becomes:

$$\hat{E\bar{N}} \simeq \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\hat{B}_{ij} - dK_{ij}}{V_{ij}}$$

where

$\hat{E\bar{N}}$ is the expected value of the rate of return from individual investments made at the margin for all parcels

$\hat{B}_{ij}$ is the estimated benefit

dK_{ij} is the cost of the investments at all locations distributed over time

V_{ij} is the total value of the investments at all locations

n is the number of potential investors for a particular parcel

m is the number of parcels available for investment

It then follows that $\hat{E\bar{N}}_{max} \geq \hat{E\bar{N}} \geq 0$ then $\lim_{\bar{N} \to 0} \hat{E\bar{N}}_{max} = \hat{E\bar{N}}$ providing that

$$V_{ij} \geq dK_{ij} \quad \forall \; K_{ij}$$

Having considered the player investment in detail, one can now develop this process in terms of the general computer equations. At any particular time period, the possibility that the net rate of return is at any point on the long or short continuums can be expressed as:

$$\bar{N} = \sum_{i-1}^{n} N_i P_i \qquad P_i = P(N = N_i)$$

where

$\bar{N}$ is the net rate of return on the chosen investment

P_i is the probability function

It then follows from the definition of expected value that:

$$\bar{N} = \sum_{i=1}^{n} d_i P_i = E\bar{N}$$

where

d_i is a specific decision by player i

Hence, $E\bar{N}$ can be viewed as the expectation of a particular parcel and $\hat{E\bar{N}}$ given earlier can be viewed as an estimator for total investment in a particular round. The computer simply views the equation:

$$\hat{E\bar{N}} \simeq \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\hat{B}_{ij} - dK_{ij}}{V_{ij}}$$

as

$$E\bar{N}_j = \frac{\sum_{p=1}^{m_j} \hat{B}_p - \sum_{p=1}^{m_j} C_p}{\sum_{p=1}^{m_j} V_p}$$

where

$E\bar{N}_j$ is the expected rate of return for an investment of a given type

$\hat{B}_p$, C_p, V_p as defined before for a particular parcel

m_j is the number of parcels of a given type

The next chapter deals with the various types of investments available to the players. It should be noted that when a player makes his investment decisions, he places these decisions on a coding form for computer processing. The computer in turn acts only as an accountant and does not judge the merits of any investment. If the investment is possible under the particular game constraints, it will be allowed. The statistics published by the

computer for total investments are merely the summation of investments successfully made during a particular round.

C. Generalized Private Capital Allocation Equation

Private capital is invested in fixed assets through three general categories residential housing, commercial activities and industrial plant and equipment. Since each of these categories can be subdivided in types of land development and no two types of land development can occupy the same parcel of land, we can write the general equation of a particular type of land use summed over all parcels for the whole system. Thus

$$\bar{N}_j = \sum_{p=1}^{m_j} \left\{ \frac{\hat{B}_p - C_p}{V_p} \right\}$$

where

$\bar{N}_j$ is the expected rate of return for all investments of a particular type j in the whole system

$\hat{B}_p$ is the benefit or income for a particular parcel's land use

C_p is the cost discounted over time for a particular parcel's land use

V_p is the value of the investment on a particular parcel

m_j is the total number of parcels of a given type of land use investment

This equation then provides the expected rate of return throughout the system for a particular type of investment. The following is a detailed analysis of this type of capital investment in terms of the expected rate of return.

The three general categories of fixed assets are subdivided as follows:

1. Residential Housing

   a) Single Family Dwelling
   b) Duplex and Garden Apartments
   c) Multiple Units and High Rise Apartments

2. Commercial Activities

   a) Personal Goods
   b) Personal Services
   c) Business Goods
   d) Business Services

3. Industrial Plant and Equipment

   a) Manufacturing

      1) Furniture and Lumber
      2) Stone, Clay and Glass
      3) Primary Metals
      4) Fabricated Metals
      5) Non-Electrical Machinery
      6) Electrical Machinery
      7) Transportation Equipment
      8) Food
      9) Textiles, Apparel and Leather
      10) Paper
      11) Chemical, Plastics and Rubber

   b) Non-Manufacturing

      1) National Services
      2) Construction Industry

The balance of this section deals with the development of the equations for these general categories. The industrial category will be treated as two categories: Manufacturing and Non-Manufacturing.

In each of the three major categories, we must develop equations for income, expenditures and value in order to apply $\bar{EN}_j$.

D. Residential Housing

The income for a given residential property is the product of the amount of space used and the rent per space unit for the property. This is expressed as:

$$\hat{B}_p = R_p \cdot S_p$$

where

R_p is the rent per space unit

S_p is the number of space units occupied

Let

$$S_p = \sum_c T_{p_c} \cdot K_c$$

where

T_{p_c} is the number of population units of class c on property

K_c is a conversion factor for the class

Since we are summing over all available classes, i.e., high, middle and low income classes in each parcel of land, S_p may be substituted into the equation $\hat{B}_p = R_p \cdot S_p$. When this substitution is made, you get:

$$\hat{B}_p = R_p \left(\sum_c T_{p_c} \cdot K_c\right)$$

And summing over all parcels we get total income

$$\sum_{p=1}^{m_j} \hat{B}_p = \sum_{p=1}^{m_j} R_p \left(\sum T_{p_c} \cdot K_c\right)$$

The expenditures for a property (C_p) are found by summing for each property the utility, tax, maintenance and water costs. This can be expressed for a single parcel as:

$$C_p = UT_p + TX_p + MT_p + WTR_p$$

where

UT_p is the parcel utility expenditure

TX_p is the total parcel tax expenditure

MT_p is the maintenance cost

WTR_p is the water cost

TX_p, the total parcel tax expenditure, is the sum of three taxes: property, income and sales taxes.

The residence's maintenance expenditure is the sum of its expenditures for personal goods and personal services. The number of consumption units required for maintenance is a function of the total percent depreciation, the level of the residence, and the number of PG and PS units required for each percent depreciation. Total percent depreciation is the sum of normal depreciation and depreciation due to other factors. $D_T = D_N + D_O$. D_N is obtained from the Master Tables,[1] and D_O is due to over use and abuse of the residence. This is usually provided on the residence output. Note that the Master Table also provides the amount of PG and PS units required for each 1% of total residence depreciation. Let M_{PG} and M_{PS} be the amount required for each percent depreciation. Thus with a level of residence at three, the amount PG and PS units required are:

$D_T \cdot M_{PG} \cdot 3$ in PG units of consumption

$D_T \cdot M_{PS} \cdot 3$ in PS units of consumption

The price that the residence owner pays depends on where he purchases PG and PS and this is obtained from the Commercial Detail Output.

Since we are interested in the cost for all parcels, C_p is merely summed over all parcels to get:

---

[1] RMB Economic Sector Manual P. 129.

$$\sum_{p=1}^{m_j} C_p = \sum_{p=1}^{m_j} (UT_p + TX_p + MI_p + WTR_p)$$

Finally we must calculate the value (V_p) to complete $\tilde{N}_j$. The value of a property is the sum of the value of the land occupied and the value of the structure.

The value of the occupied land depends on the level of development of the three types of residential development. The term levels refers to level of development and not the condition of the structure. It is important to remind ourselves that when we refer to a level 1 (e.g., RB1 or a duplex type of dwelling at one level of development) we are in aggregate terms and for this example we are really talking of a type of housing which provides dwellings for six to twelve thousand people distributed throughout the parcel or square mile. In other words we are not talking about a single duplex building but instead we are referring to a large group of buildings aggregated by type. Since no two types could occupy the same parcel even among housing, we cannot have a RA and RB type in the same parcel. In addition, since we know that each of the three types of housing only occupies two percent of the land at a level one development, we may develop each type to a maximum of level fifty which would occupy one hundred percent of the land. A RB developed to a level of fifty would provide housing for fifty times the population density of 300,000 to 600,000 persons. Thus the product of level development, the space required for each type of housing and the unit market value of the land on the parcel is the value of the occupied land. This value of occupied land can be expressed as:

$$LND_p = L_p \cdot S_h \cdot U_p$$

where

LND_p is the value of the occupied land on parcel p

L_p is the number of levels of property development

S_h is space in units of percentage of the unit value of land required for a single level of development of type h (h = RA, RB, RC) where RA, RB, RC refer to the types of single, duplex or multiple dwellings

U_p is the unit value of land at parcel p (e.g., LND_p for an RB3 = $3 \cdot \frac{2}{100} \cdot 635{,}000 = \$38{,}100$)

The structure value for a given property is the original cost of a level of the structure (type specified) times the value ratio times the number of levels on the parcel. The structure value may be expressed as:

$$BLDG_p = L_p \cdot O_h \cdot VR_p$$

where

$BLDG_p$ is the structure value of parcel p

L_p is the number of levels of property development

O_h is the original cost of a level one development of a structure of type h

VR_p is the value ratio of building on parcel p

As earlier stated, V_p for a particular parcel is equal to the sum of the value of land and buildings or

$$V_p = LND_p + BLDG_p$$

$$V_p = L_p \cdot S_h \cdot U_p + L_p \cdot O_h \cdot VR_p$$

$$V_p = L_p (S_h \cdot U_p + O_h \cdot VR_p)$$

Summing over all parcels of land we have

$$\sum_{p=1}^{m_j} V_p = \sum_{p=1}^{m_j} L_p (S_h \cdot U_p + O_h \cdot UR_p)$$

Finally substituting into $\tilde{N}_j$ we have:

$$\bar{N}_j = \frac{\sum_{p=1}^{m_j} R_p \left(\sum_c T_{pc} \cdot K_c\right) - \sum_{p=1}^{m_j} (UT_p + TX_p + MT_p + WTR_p}{\sum_{p=1}^{m_j} L_p (S_h \cdot U_p + O_h \cdot VR_p)}$$

E. Commercial

The income for any commercial establishment is simply the product of the number of units sold and the price per unit or:

$$\hat{B}_p = US_p \cdot P_p$$

where

US_p is the units sold

P_p is the price per unit

As before, summing over all parcels results in:

$$\sum_{p=1}^{m_j} \hat{B}_p = \sum_{p=1}^{m_j} US_p \cdot P_p$$

The expenditures for a given commercial property are simply the sum of the utility, tax, maintenance, water, salary, transportation, and goods and service costs. This is written as:

$$C_p = UT_p + TX_p + MT_p + WTR_p + SL_p + TR_p + GS_p$$

where

C_p, UT_p, TX_p, MT_p and WTR_p are defined as before

SL_p are salaries paid

TR_p are transportaiton costs

GS_p is the cost of goods and services

It is important to note that the price is set by the seller or owner of the commercial plant. The buyer who requests the number of units he

needs is granted the units if the seller can supply them. The commercial establishment is either business commercial (BG and BS) or personal commercial (PG and PS). BG and BS purchase their needed supplies from the outside systems whereas PG and PS purchase their supplies from the local BG and BS establishments. The customers of BG and BS are either private (i.e., PG, PS and Industrial establishments) or public (i.e., Municipal Services Highway Department, etc.).

BG and BS spend money for service charges, GS_p, which represent the purchases from the outside system. On the other hand PG and PS spend money for business goods and business services whigh are together referred to as GS_p. This GS_p is the sum of goods and services that are required for the PG and PS to operate. In both cases for BG, BS and PG, PS, GS_p is directly related to the number of capacity units sold. Hence the model assums that the commercial establishments only purchase as much as is required for them to produce the number of capacity units sold. This relationship is found in the Master Table for commercial business. The production of 1 CU (capacity unit) of PG requires the purchase of .037 CU of BG and .017 CU of BS.

Transportation costs incurred by commercial business are dependent upon the number of capacity units purchased, the type of buyer and seller, the distance travelled to the destination and the type of roads used. Regardless of the distance travelled, a business pays a base cost to travel to a destination. Thus the total transportation cost can be expressed as follows.

$$TR_p = (CU_p \cdot BTC) + \{CU_p \cdot \bar{BTC} \cdot L \cdot [(4 \cdot NT_p) - RT_p]\}$$

where

CU_p is the number of capacity units consumed

BTC is the base transportation cost per unit consumed

thus $(CU_p \cdot BTC)$ is the base transportation cost to business

and L is the length of a parcel side in miles for this model

NT_p is the number of parcel sides travelled along the least cost route between origin and destination

RT_p is the sum of the road types traversed along the least cost route

$\bar{BTC}$ is the transportation cost per unit per mile travelled and it is numerically the same as BTC

hence $[(4 \cdot NT_p) - RT_p]$ is the weighted sum of parcel sides travelled from origin to destination along least cost route. The parcel sides or roads are weighted according to the road quality; the best road being of type 3 and the worst being of type 1. Thus four parcel sides of type 2 produce a weighted sum of (16 - 8) = 8 parcel units. Hence if each parcel side is 1 mile long, then the weighted distance travelled is 8 mile units and the cost with a $\bar{BTC}$ of \$400 per mile per unit for 3000 units is:

$$TR_p = (3000 \cdot 400) + \{3000 \cdot 400 \cdot 8\} = \$10,800,000$$

Note BTC and $\bar{BTC}$ are obtained from the Commercial Master Table.

Again summing over all parcels of land you have:

$$\sum_{p=1}^{m_j} C_p = \sum_{p=1}^{m_j} (UT_p + TX_p + MT_p + WTR_p + SL_p + TR_p + GS_p)$$

Finally the value of the property is calculated in the same manner as housing or

$$V_p = L_p (S_h \cdot U_p + O_h \cdot VR_p)$$

or summed over all parcels:

$$\sum_{p=1}^{m_j} V_p = \sum_{p=1}^{m_j} L_p (S_h \cdot U_p + O_h \cdot VR_p)$$

Substituting the values of $\hat{B}_p$, C_p and V_p into the general formula for $\bar{EN}_j$ we have for commercial investment:

$$E\bar{N}_j = \frac{\sum_{p=1}^{m_j} US_p \cdot P_p - \sum_{p=1}^{m_j} (UT_p + TX_p + MT_p + WTR_p + SL_p + TR_p + GS_p)}{\sum_{p=1}^{m_j} L_p (S_h \cdot U_p + O_h \cdot VR_p)}$$

F. Industrial

The income for a given industrial property is the product of the price per unit (industry specific) and the number of units produced on the properties. This is defined as:

$$\hat{B}_p = P_m \cdot UP_p$$

where

P_m is the industry specific price

UP_p is the number of units produced

UP_p for all industrial property is at a maximum of 1000 units per level. The actual number produced depends on the employment effect and the value ratio of the property.

Thus

$$UP_p = EE_p \cdot VR_p$$

where

EE is the employment effect

and

VR is the value ratio of the property , i.e., 90/100 or .9

The value ratio is determined by the amount of depreciation that occurred during a given round but it is limited by the level at which maintenance is set. So that if the maintenance level is set at 70% then the value ratio cannot fall below this level of 70%. In fact, maintenance is automatically done each year to keep the property at this value ratio.

The employment effect, EE, is the product of the maximum employment effect, 1000, and the fraction of the total number of employees hired to

the total number of employees required.

$$EE = EE_{max} \cdot EH/ER$$

where

EE_{max} are the maximum employment effect

EH are the employees hired

ER are the employees required

P_m, the price per unit of industrial product is determined by the national business cycle price relative and this normal price per unit for the industrial type.

Thus

$$P_m = NBCPR \cdot NP_m$$

NBCPR is the national business cycle price relative which is generated by the outside system i.e., the computer. Its range is .90 to 1.12. In each round, the current NBCPR is printed under the Transactions with the National Economy.

NP_m is normal price per unit of the industrial type sold and is obtained from the Master Table for Industrial Establishments for each type.

Summing over all industries properties of a type:

$$\sum_{p=1}^{m_j} \hat{B}_p = \sum_{p=1}^{m_j} P_m \cdot UP_p = P_m \sum_{p=1}^{m_j} UP_p$$

The expenditures for an industrial property are the same set as those for commercial properties (UT_p, TX_p, MT_p, NTR_p, TR_p, SL_p, GS_p) plus terminal usage costs (TM_p) and expenses connected with pollution control (PTN_p). As before the expenditures are found by summing these values for each parcel and then summing for all parcels to get:

$$\sum_{p=1}^{m_j} C_p = \sum_{p=1}^{m_j} (UT_p + TX_p + MT_p + WTR_p + TR_p + SL_p + GS_p + TM_p + PTN_p)$$

The value of industrial property is the same formula as for commercial property, namely:

$$\sum_{p=1}^{m_j} V_p = \sum_{p=1}^{m_j} L_p(S_h \cdot U_p + O_h \cdot VR_p)$$

Substituting these values into $E\bar{N}_j$ we have:

$$E\bar{N}_j = \frac{Pm \sum_{p=1}^{m_j} UP_p - \sum_{p=1}^{m_j} (UT_p + TX_p + MT_p + WTR_p + TR_p + SL_p + GS_p + TM_p + PTN_p)}{\sum_{p=1}^{m_j} L_p(S_h \cdot U_p + O_h \cdot VR_p)}$$

III. GOVERNMENT CAPITAL

The main objective for the government is to maximize total benefits to the system by the allocation of funds among the various departments. Before we can consider an allocation scheme, we must first define the sources of income available to a governmental jurisdiction in the game. The government income is derived from property taxes, income taxes, sales taxes, auto expenditure taxes, bonds (2 year short-run and 25 year long-run), charges for services and transfers of funds, i.e., fines, bribes, etc.. It should be noted that aid from state and federal sources are not treated as income but rather as a reduction in the cost of the local projects they support.

Having considered the sources of income, we can now express the supply of allocatable funds for a jurisdiction (MG_j) as:

$$MG_j = \sum_{i=1}^{4} (TX_i)_j + B_j + C_j + TR_j$$

where

$$\sum_{i-1}^{4} (TX_i)_j = TX_1 + TX_2 + TX_3 + TX_4$$

and

TX_1 are property taxes

TX_2 are income taxes

TX_3 are sales taxes

TX_4 are auto expenditure taxes

B_j are bonds (long and short)

C_j are charges for services

TR_j are transfers from all other sources including the outside system

A. Government Decision Objectives

The main objective for the government is to maximize total benefits to the system through the allocation of funds among the various departments. If one considers the money supply available for each jurisdiction (MG_j), a limit is obviously placed on the supply of money available for distribution. Each governmental department places a demand on this supply which suggests a balanced budget to be one where supply equals demand. To quantify this idea, consider for each department, or r_d which is the ratio of demand to supply for services provided by department d, or:

$$r_d = \frac{D_d}{S_d}$$

where

D_d is the demand for departmental services

S_d is the supply of services by department d

Since these services are directly related to the money available, one can talk about services as well as money. For instance, to reduce the student-teacher ratio you must either build a new school or hire more teachers since its not realistic to reduce the pupils unless there is heavy out migration. Both of these solutions reduce taxable income which directly relates services available to money available for services in each governmental department.

The objective for each department is to bring r_d to 1.00 from either side for a balanced budget. If $r_d > 1.00$, then $D_d > S_d$ and there is not a sufficient supply of services from department d. Conversely, if $r_d < 1.00$ then $D_d < S_d$ and the supply of services from department d is in excess of the demand for those services and a wasteful condition exists.

If one now assumes that the overall government ratio is a function of all department ratios, then it is possible to determine an overall set of Δr_d based or estimated changes in supply and demand, $\Delta\hat{r}d$, which will bring the overall government ratio to 1.00. In other words, the

total demand for government services and the supply of all government services are balanced.

R_d be the overall government ratio and

$$R_d = \frac{\sum_{d=1}^{n} w_d \, r_d}{\sum_{d=1}^{n} w_d}$$

where

r_d is a departmental ratio

w_d is a weighing factor

In this case, $R_d = 1.00$ when all $r_d = 1.00$, or when tradeoffs (as determined by the w_d) balance. When $R_d > 1.00$, the total demand for services exceeds the supply and when $R_d < 1.00$, the supply exceeds the demand and a wasteful condition exists.

Hence the objective function is to bring R_d as close as possible to 1.00 with the constraints of a finite amount of allocatable funds MG_j and that all individual departments meet as much of their estimated demands without exceeding the amount of money required to finance their decisions.

B. Departmental Ratios

1. The ratio for education (I_{sc}) can be expressed in terms of supply and demand as:

$$I_{sc} = \frac{D_{sc}}{S_{sc}} = \frac{\sum_{c=1}^{n} P_c \cdot SP_c}{\sum_{p=1}^{m} SC_p \cdot VR_p \cdot T_p}$$

where

P_c is the population by class

SP_c are the students by class

SC_p is the school facility by parcels

VR_p is the value ratio

T_p is the capacity as a function of teacher mix

2. The ratio for adult education (I_{AE}) can be expressed as:

$$I_{AE} = \frac{D_{AE}}{S_{AE}} = \frac{\sum_{c=1}^{n} \sum_{p=1}^{m} EF_{cp}}{\sum_{p=1}^{m} Ph_p + Pm_p}$$

where

EF_{cp} is the time allocated to free adult education by parcel and class

Ph_p are the PH's hired

Pm_p are the PM's hired

3. The ratio for municipal services (I_{ms}) is:

$$I_{ms} = \frac{D_{ms}}{S_{ms}} = \frac{\sum_{p=1}^{m} LU_p \circ L_p \bullet MU}{\sum_{p=1}^{m} MS_p \circ VR_p \bullet T}$$

where

LU_p is the land use

L_p is the number of levels on property

MU is the MS units demanded per level of land use

MS_p is the number of facilities per parcel

VR_p is the value ratio per parcel

T is capacity as a function of worker mix

4. The ratio for welfare payments (I_{WL}) is:

$$I_{WL} = \frac{\text{Actual WL payments}}{\text{Desired or estimated WL payments}}$$

5. The ratio for utilities (I_{UT}) is:

$$I_{UT} = \frac{D_{UT}}{S_{UT}} = \frac{\sum_{p=1}^{m} LU_p \cdot L_p \cdot UU}{\sum_{p=1}^{m} UT_p \cdot DZ}$$

where

UU equals UT units demanded per level of land

UT_p is the utility of a plant facility on a parcel

DZ is the optimal operational capacity of a utility plant

LU_p is the land use

L_p is the number of levels of development on property

6. The ratio for transportation (I_{TR}) is:

$$I_{TR} = \frac{D_{TR}}{S_{TR}} = \frac{(\alpha\beta\gamma) \sum_{i=1}^{n} PW_i + \beta \sum_{p=1}^{m} \sum_{i=1}^{n} (PS_i + L_i)_p}{LRC}$$

where

α is the average travel time

β is the average cost

γ is the average inconvenience

$\alpha\beta\gamma$ is a measure of the mean ecological distance

PW_i are workers

PS_i are shops

L_i is the inter-business transport

P is parcel

LRC is the lowest real cost. This cost is at a minimum when $\alpha = \alpha_p$, $\beta = \beta_p$ and $\gamma = 0$

C. Budget Allocation

The total budget to be allocated to each department must be the entire supply in order to avoid wastefullness. This decision format for allocation can be expressed as:

$$MG_j \simeq \max \sum_{\ell=1}^{n} (IG_\ell + \zeta + P_i)$$

$$\text{s.t. } IG_\ell \geq 1$$

where

IG_ℓ are the specific services indexes for each department

ζ is a random factor for deciding expenditures

P_i is the sum of all public and private influence which affect allocation decisions

Since the objective is clearly to spend all available funds, i.e., $IG_\ell \geq 1$, the tendency will be to allocate the government capital resources to departments with the highest ratios. This is the reason for optimizing with a maximum.

IV. TIME ALLOCATION

The social sector directly influences and controls the government through its voting power and the economic sector through its ability to boycott. In addition, it has the ability to allocate the leisure time of small aggregates of workers which produce complex interactive effects throughout the entire model. Leisure time is that time not spent in full-time work, eating, sleeping and personal care. It is divided into 100 units which can be allocated to part-time work, adult education, political activity or recreation. In addition, the time used to travel to and from work is subtracted from the leisure time. A further reduction of leisure time occurs due to sickness as a result of the high health index at a workers resident location. With this in mind we would describe the total amount of allocatable time, T, as:

$$T = Tu \bullet bPk$$

where

T is the total amount of real time units of leisure time used in the entire system

Tu is the specific amount of time per unit

bPk is the total number of people in the system by socio-economic class weighted by the real value of their time units (b)

In theory, this model is sufficient, but in practice it takes into account such detail that the model would become extremely complicated.

To avoid this difficulty, define the real time available as:

$$T = (T_N + T_A)bPK$$

where

T_N is the fixed amount of time that is more or less available and falls outside short-run discretionary allocations

T_A is the amount of real time actually available to be allocated in the short-run

A. Allocations

Let T_N be defined as before and

$$T_N = T_T + T_Z + T_I$$

where

T_T is the time spent in traveling to work

T_Z is time lost to illness

T_I is time spent in involuntary activities

Using the above definitions, T_A can then be considered as resource improving uses of time which can be controlled as time allocations.

Let

$$T_A = T_W + T_{AP} + T_{AF} + T_{PA} + T_R$$

where

T_W is time spent in a part-time job

T_{AP} is time spent in paid training and education programs

T_{AF} is time spent in free training and education programs provided by the local school system

T_{PA} is time spent in political activities

T_R is time spent in recreational pursuits

Associated with each of the pursuits in T_A is a rate of return for the time invested given by:

1. $$r_w + \gamma_1 \left(\frac{SAL_c}{N_c} \right)$$

where

r_w is the rate of return from part-time work

SAL_c is the full-time salary offered to the class c

N_c is the relative value of part-time work to class c

γ_1 is a function designed to make the absolute value of r_w relative to other time pursuits for comparison and is defined as:

$$\gamma_1 = f \left(\frac{\sum_{c=1}^{m} P_c}{\sum_{c=1}^{m} JOB_p t_c}, \text{IND.} \right)$$

where

P_c is the population units by class wishing to allocate time units to part-time jobs

JOB_{Pt_c} are part-time jobs offered by all employees for a class c (of class M & L)

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

2. $\gamma AF_c = \gamma_2(\Delta EL_c)$

where

γAF_c is the rate of return for free training and adult education

ΔEL_c is the change in education level for class c

γ_2 is a function designed to make the absolute value of γAP_c relative to other time pursuits for comparison and is defined as:

$$\gamma_2 = f\left\{\frac{\sum_{j=1}^{m} P_{ML_j}}{\sum_{i=1}^{n} FAE_i}, IND\right\}$$

where

P_{ML_j} is the number of population units wanting to allocate time to free training and education

FAE_i is the supply of free education and training programs provided by the local school systems

IND. is an indeterminate quantity such as prosperity, likes, dislikes, etc.

3. $\gamma AP_c = \gamma_3 (\Delta EL_c - PR_c)$

where

γAP_c is the rate of return for paid training and education

ΔEL_c is the change in educational level for class c

PR_c is the cost of spending one-time unit in private education and training

γ_3 is a function designed to make the absolute value of r_{AFC} relative to other time pursuits for comparison and is defined as:

$$\gamma_3 = f(C_{AE}, \text{IND.})$$

where

C_{AE} is the cost of private training and education

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

4. $r_{PC} = \gamma_4 \Delta VT_c$

where

r_{PC} is the rate of return for political activities

ΔVT_c is the increase in voter registration for class c

γ_4 is a function designed to make the absolute value of r_{PC} relative to other time pursuits for comparison and is defined as:

$$\gamma_4 = f(rP_p, \text{IND.})$$

where

rP_p is the expected return from political power

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

5. $\gamma R_c = \gamma_5 \Delta PS_c$

where

γR_c is the rate of return for recreational activities

ΔPS_c is the decrease in personal dissatisfaction due to recreation for class c

γ_5 is a function designed to make the absolute value of γR_c relative to other time pursuits for comparison and is defined as:

$$\gamma_5 = f\left\{\frac{\sum_{j=1}^{m} P_{RT_j}}{\sum_{i-1}^{n} PLA_i}, \text{IND}\right\}$$

where

P_{RT_j} is the number of population units wanting to allocate recreational time

PLA_i is the supply of park land and other recreational facilities usually provided by the planning and zoning department

IND. is an indeterminate quantity

B. Decision to Invest Time

Let $\tilde{T}$ be the pursuit one choses to invest time in given some return rate from $\tilde{T}$. Then:

$\tilde{T} = T_W$ if $r_W \geq r_{APC}, r_{AFC}, r_{PC}, r_{RC}$

$\tilde{T} = T_{AP}$ if $r_{APC} \geq r_W, r_{AFC}, r_{PC}, r_{RC}$

$\tilde{T} = T_{AF}$ if $r_{AFC} \geq r_W, r_{APC}, r_{PC}, r_{RC}$

$\tilde{T} = T_P$ if $\mathbf{r_p} \geq r_W, r_{APC}, r_{AFC}, r_{RC}$

$\tilde{T} = T_R$ if $r_{RC} \geq r_W, r_{APC}, r_{AFC}, r_{RC}$

Further, if $\tilde{r}$ is the return rate from $\tilde{T}$ and B is a benefit from all time allocatable investments, then:

$$B = \sum_{i=1}^{n} \tilde{r}_i \tilde{T}_i$$

where

$$T_i \varepsilon \{T_w, T_{AP}, T_{AF}, T_P, T_k\}$$

An investor in allocatable time will tend towards maximizing B by finding the best mix of time investments according to:

$$\max B = \max \sum_{i-1}^{n} \tilde{r}_i \tilde{T}_i$$

V. PUBLIC AND PRIVATE INFLUENCE

Public and private influence is by far the most difficult to capture in quantitative form. It is desirable to include this so called "power" into the game since voting, economic suasion, and influence peddling in the form of bribes and boycotts are all part of a real city. For the purposes of the game, this quantity must be in the standard form of supply and demand recognizing that government officials vote on issues and private sector members vote for government officials. This dual role can be readily seen to apply in the economic and influence roles as well. It is within this dual functional idea, that this model is built. One could for instance, claim that public influence is related to legal cases it wins, the percentage of expenditures for influencing voter groups and commercial enterprises and the moral suasion of the incumbents. On the other hand, private power can also take a large number of forms. We could include voting power, lobbying, migration, bribes and numerous other influences such as protests that influence government decisions.

The ratios for economic power were described in Chapters II and III and have the built-in bias of determination for public and private goals. It is therefore convenient to express total power as:

$$P = \sum_{i=1}^{n} P_{pi} + P_{gi}$$

where

P is the total power in the system at a particular point in time

P_{pi} is the private power, both social and economic for all individuals

P_{gi} is the public power for all individuals

To develop this particular model, we must develop equations for voting, a scheme for allocating power and a supply and demand ratio in

terms of money to measure this influence so that it will be compatible with other functions of the partial resource allocation defined earlier as:

$$PR = \alpha CP + \beta CG + \delta T + \varepsilon P$$

In other words, ε must be defined explictly as well as P.

A. Voting

Voting can be described as:

$$POP_V = \sum_{k=1}^{m} \lambda_k \sum_{i=1}^{n} P_{k_i} C$$

where

POP_V is the total number of people who vote

λ_k is the propensity of each class to vote

P_{k_i} is the number of people who actually vote

C is a random factor

Since the above equation is difficult to use in its present form, we can consider the probability that a voter will vote for a particular candidate or issue. When this is done, P_k then becomes:

$$P_k = \sum_{i=1}^{n} \delta_i p_i$$

where

i is an individual member of a class

δ_i is the propensity of that individual to vote

p_i is the probability that i will vote

In other words, given the inclination to vote (δ_i), there is a probability that he will vote (p_i).

The above statements can be further described as when i is an individual, then $\forall i \exists p_i$ and $0 \leq p_i \leq 1$. Now if we let g_i be the probability that a vote will be cast for an alternative j on an issue, then $0 \leq g_i \leq 1$. When this is true, $k(p_i g_i)$ becomes the probability that i will vote for j in class k and POP_v then becomes:

$$(POP_v)_j = \sum_{k-1}^{m} \lambda_k \left[\sum_{i=1}^{n} (\delta_i p_i g_i) \right]_k$$

and finally for all issues we have

$$\sum_{j=1}^{m} (POP_v)_j = \sum_{j-1}^{m} \left\{ \sum_{k-1}^{m} \lambda_k \left[\sum_{i=1}^{n} (\delta_i p_i g_i) \right]_k \right\}_j$$

Based on the above equation, one can now define C, given as a random factor, as influence or the abstract quantity that sway voters by emotional or financial means at the last minute on issues and intended voting desires.

B. Allocation of Power

The objective sought in influencing is to allocate that maximum amount of influence in the public and private sectors. This allocation can be expressed as:

$$P \simeq \max \sum_{i-1}^{n} P_{p_i} + P_{g_i}$$

subject to:

$$P_{p_i}, P_{g_i} > 0$$

It is not reasonable to assume that a democratic government can exist without some influence from both the government and private sectors.

Based on the assumption that power exists in both sectors, one can then look at the distribution of this power between the sectors and the incremental cost associated to change this power ratio. With public and private power always in existance, we must consider incremental changes in costs for this ratio. If influence is viewed in purely capitalistic terms, there will always be a supply and demand for influence. In this format, one can now refer to ratio's in the same manner as the government ratio's of Chapter III.

Let Φ_p be defined as the incremental cost to change the ratio r_p when

$$r_p = \frac{D_p}{S_p}$$

in terms of supply and demand.

Now if $r_p = \frac{D_p}{S_p}$ we can solve for S_p and find that $Sp = \frac{D_p}{r_p}$. The change in S_p required to meet new conditions will become:

$$\Delta S_p = S'_p - S_p = \frac{D'_p}{r'_p} - \frac{D_p}{r_p}$$

when S'_p, r'_p and D'_p are the new conditions.

Rearranging the terms in ΔS_p:

$$\Delta S_p = \frac{D'_p r_p - D_p r'_p}{r'_p r_p}$$

and noting that

$$\Delta r_p = r'_p - r_p \quad \text{or} \quad r'_p = r_p + \Delta r_p$$

$$\Delta D_p = D'_p - D_p \quad \text{or} \quad D'_p = D_p + \Delta D_p$$

we can substitute these values into ΔS_p and get:

$$\Delta S_p = \frac{r_p(D_p + \Delta D_p) - D_p(r_p + \Delta r_p)}{r_p(r_p + \Delta r_p)}$$

Expanding:

$$\Delta S_p = \frac{r_p D_p + r_p \Delta D_p - r_p D_p + D_p \Delta r_p}{r_p^2 + r_p \Delta r_p}$$

Collecting terms:

$$\Delta S_p = \frac{r_p \Delta D_p + D_p \Delta r_p}{r_p^2 + r_p \Delta r_p}$$

Since the cost per unit of supply is known for each sector and is C_p, then the incremental cost to change the ratio becomes:

$$\Phi_p = C_p \Delta S_p = \frac{r_p \Delta D_p C_p + D_p \Delta r_p C_p}{r_p^2 + r_p \Delta r_p}$$

This latter equation might be viewed as the incremental cost to change the power base.

C. Power Objective

The power base can now be defined as:

$$P = V + E_p + IN$$

where

V are votes

E_p is the economic power

IN is influence

This equation may be re-written as:

$$P = \Phi_p \sum_{j=1}^{m} (POP_v)_j + E_p + IN$$

where

IN is defined in terms of money persuation and the first term is the incremental cost of swaying elections.

VI. OPERATION OF THE MODEL

The previous chapter related to the development of the partial resource allocation for the general system. In the game, the variables for the fixed and liquid resources must be related to each other to integrate all city functions. This process is accomplished through a series of operating programs plus the decision choices of the actual players. The sections of this chapter will describe the major operating programs and the calculations they perform.

A. Business Goods and Services

1. The total demand for business goods and business services is given by:

$$TD_{(BS/BG)} = \sum_{i-1}^{n} K\ell_i LU_i + C_{pg} PG_s + B_{ps} PS_s + \sum_{i=1}^{n} Z\ell_i (DEP_i + MT_i - VR_i) + G$$

where

$TD_{(BS/BG)}$ is the total demand for business goods and services

LU_i is the land use at a particular site i

PG are sales of personal goods

PS are sales of personal services

L_i are the number of levels at location i

$K\ell_i, C_{pg}, B_{ps}, Z\ell_i$ are constants

2. The supply in the commercial sector is dependent upon level, employment and value ratio given by:

$$TS = DC \sum_{i-1}^{n} L_i VR_i \left(1 - \frac{\sum_{k=1}^{m} E_{k_i}}{E^{\bullet}}\right)$$

where

TS is the total supply

DC is design capacity

L_i are the number of levels at location i

VR_i is the value ratio at i

E_{k_i} is the number of employees at i by class k

$E^{\bullet}$ is the total of employees needed at that location

3. Price is a function of the total demand and total supply curves and is given by:

$$\bar{P} = \frac{\sum_{i=1}^{n} P_i \ (t-1)}{n}$$

where

$\bar{P}$ is the average price paid last year

P_i is the price of goods or service for the last round (t-1)

n is the number of i goods or i services rendered last round

4. The new prices for goods or services are dependent on the change in demand and the change in supply for the next round.

This may be expressed as:

$$P = R \left[\frac{\Delta Po - \Delta Ps}{2} \right] \bar{P}$$

where

P is the new price for goods or services

ΔPo is the change in price if supply is kept constant

ΔP_s is the change in price if demand is kept constant

R is a functional multiplier

$\overline{P}$ is the average price paid last year

B. Employment

The employment process matches the people who are looking for jobs with the employers who are trying to obtain workers for needed skill levels.

1. Demand for Jobs

The demand for jobs is a function of the amount of new industry in the system, the expansion of government services, the displacement of workers from firms going out of business or site location changes of existing firms. In the game, the demand is also caused by jobs not filled from the previous round.

This demand for jobs is:

$$J_{jk} = NP_k + NG_k - J_k + V_k$$

where

J_{jk} is the total demand for jobs by each income class

NP_k is the amount of new industry constructed during the previous time period

NG_k are the new government positions

J_k are jobs created due to employee displacement

V_k are job openings from the previous round.

2. Supply of Workers

The supply of workers is given by:

$$P_{jk} = P_{sk} + P_{nk} + PL_k + P_{xk}$$

where

P_{jk} is the total worker supply by socio-economic class

P_{sk} are workers who desire to move by skill category

P_{nk} are new residents in the system

PL_k are people who have moved in the system or have otherwise vacated their original jobs

P_{xk} are the remaining unemployed persons

3. Matching

The process of matching the supply and demand for workers is accomplished by locating workers by skill and educational level to fill the demand,whereas the workers are interested in obtaining positions which maximize that income and/or achieve their satisfying objectives. To accomplish this matching process, the supply (P_{jk}) is ranked by educational level, the most skilled first in the form:

$$P_{jkp} \max(S_{epk} + KS_{ep} - T_{epp'})$$

if

$$\sum_{k=1}^{m} P_k > \sum_{k=1}^{m} J_K$$

then

$$P_{jkp} = \max [S_{ep'(k-1)} + KS_{ep'(k-1)} - T_{epp'}]$$

where

P_{jkp} is the supply of jobs by parcel and class

$S_{ep'k}$ is the salary offered on parcel position

$T_{epp'}$ is the transportation cost to travel from home to work

K is the percent change in job attractiveness to entice worker to change positions, i.e., salary, etc.

This process continues for each class until

P_k matched with J_k are employed

P_k matched with J_{k-1} are unemployed

P_k not matched are unemployed

4. Part-Time Employment

Part-time jobs must be matched against workers who desire part-time work. The matching is performed in the same manner as for full-time work, namely:

$$SPT_k = \sum_{i=1}^{n} Te_i \; Z$$

where

$$Z = \sum_{k=1}^{m} P_k$$

and

SPT_k is the supply of part-time jobs by parcel

Te_i is the transportation cost per individual

Z is the sum of available jobs by parcel

5. Salaries

The salary that can be offered at a given location varies with transportation costs, unemployment and salaries offered at other locations. Salary may be computed according to the formula:

$$S\phi = \bar{S}BZ_k U\left[\frac{\bar{T}\phi i}{\bar{T}}\right]$$

where

$$\bar{S} = \sum_{j=1}^{m} S_j E_j$$

$$U = \frac{\sum_{j=1}^{m} E_j}{Z_k}$$

$$\bar{T}\phi = \frac{\sum_{i=1}^{m} T_i}{Z_k}$$

$$\bar{T} = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n} T_{ij}}{\sum_{j=1}^{m} E_j}$$

and upon substitution

$$S\phi = \left[\sum_{j=1}^{m} S_j E_j\right] BZ_k \left[\frac{\sum_{j=1}^{m} E_j}{Z_k}\right] \left[\frac{\sum_{i=1}^{n} T_i}{Z_k} \Big/ \frac{\sum_{j=1}^{m}\sum_{i=1}^{n} T_{ij}}{\sum_{j=1}^{m} E_j}\right]$$

$$S\phi = \left[\sum_{j=1}^{m} S_j E_j\right] BZ_k \left[\frac{\sum_{j=1}^{m} E_j}{Z_k}\right] \left[\frac{\sum_{i=1}^{n} T_i \sum_{j=1}^{m} E_j}{Z_k \sum_{j=1}^{m}\sum_{i=1}^{n} T_{ij}}\right]$$

$$S\phi = \frac{BZ_k \sum_{j=1}^{m} E_j \sum_{j=1}^{m} S_j \sum_{i=1}^{n} T_i}{Z^2_k \sum_{j=1}^{m}\sum_{i=1}^{n} T_{ij}}$$

$$S\phi = \frac{B \sum_{j=1}^{m} E_j S_j \sum_{i=1}^{n} T_i}{Z_k \sum_{j=1}^{m} \sum_{i=1}^{n} T_{ij}}$$

where

$S\phi$ are the total salaries

E_j are the number of employees at j

S_j is the salary per worker at j

T_i is the transportation cost for work i

Z_k is the total labor force by class

B is the coefficient of unemployment

C. Land Use

In the previous section concerning the allocation of private capital sources, we assumed that an allocation could be made for a particular land use type. In general, if L_{ij} is the cost of locating a specific land use in a particular location, then:

$$\text{Land Costs} \simeq \sum_{i=1}^{n} \min L_{ij}$$

subject to:

Formal Zoning
Presenc of Utilities
Topographical Features

and

$$Y_d \geq B_c + D + LC$$

where

Y_d is the discounted income

B_c is the structure cost

D is the demolition condition

LC is the land cost

One can further require that

$$MS_L \geq B_c + D + LC$$

and

$$G_y = k_i$$

where

MS_L is the money supply available for land

G_y is a constant gross income per land use

Within this framework a model for specific land uses can be developed.

1. Residential

Residential land use can be modeled as:

$$L_R \simeq \sum_{i=1}^{n} \min L_{ij} = \alpha_1 EI_i + \alpha_2 LC_i + \alpha_3 \bar{T}_i$$

where

EI_i is an environmental index for a parcel

LC_i is the parcel land cost

$\bar{T}_i$ is the average transportation cost

α_1 α_2 α_3 are coefficients to relate the variables to each other

2. Industrial

Industrial land use is:

$$L_I \simeq \sum_{i=1}^{n} \min L_{ij} = \beta_1 \bar{T}_i + A$$

where

$$A = \beta_2 LC_i + \beta_3 CH_i + \beta_4 T_i + \beta_5 W_i + \beta_6 MS_i$$

and

$\bar{T}_i$ is the average transportation cost

LC_i is the parcel land cost

CH_i are the utility charges

T_i are the taxes

W_i are the water charges

MS_i are the municipal service charges

β_i are coefficients to relate the variables to each other

3. Commercial

The commercial land use model is:

$$L_C \simeq \sum_{i=1}^{n} \min L_{ij} = \gamma_1 A_i + \gamma_2 B_i$$

where

A_i is the same as industrial

$B_i = \max \sum_{i=1}^{n} C_{ij}$

or

B_i is the best relative competative advantage for a new location given all existing land uses

γ_1, γ_2 are coefficients to relate the variables to each other

4. Government Facilities

$$Lg \simeq \max B = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{ij}$$

given:

The existing government land uses (Lg_{it})

Location of all other land uses (LU_i)

Location of all people by class (P_{ik})

and

N_{ij} are any government services to be located in a specific jurisdiction

D. Land Value

Land value is a function of the proximity of the particular parcel of land to various facilities and the presence of certain amenities. In equation form this becomes:

$$LV = BV + \varepsilon_1 T_T + \varepsilon_2 R + \varepsilon_3 E + Z + \varepsilon_4 T_R$$

where

LV is the land value of a particular parcel

BV is the base value which is usually the agricultural value or approximating the construction value

T_T is the distance to the nearest terminal

R is the distance to the nearest residence

E is the distance to the nearest employment opportunity

Z is the presence of zoning by type

T_R is road access by type of road

E. Rent

The rent for a particular dwelling can be calculated according to the following formula:

$$R_f = (\bar{R}) \left(\frac{BQI_f}{\bar{QI}} \right) \left(\frac{\gamma \bar{E}\bar{I}}{EI_f} \right) (\bar{\phi}_c)$$

where

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i S_i}{\sum_{i=1}^{n} S_i}$$

$$\bar{QI} = \frac{\sum_{i=1}^{n} QI_i}{n}$$

$$\bar{EI} = \frac{\sum_{i=1}^{n} EI_i}{n}$$

$$\bar{\phi}_c = \frac{\sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} Z_i}$$

and

$\bar{R}$ is the average rent over system

R_i is rent per space unit

S_i is space units rented

QI_i is quality of housing index

EI_i is environmental index

$\bar{QI}$ is average quality of housing index

$\bar{EI}$ is average environmental index

$\bar{\phi}_c$ is average occupancy

Z_i is total space units available at parcel i

QI_f is the quality housing index for a particular dwelling

EI_f is the environmental index for a particular dwelling

F. Location of a Commercial Site

In order to locate any non-residential income producing property, one must consider:

1. Cost-distance to the nearest terminal
2. Cost-distance to suppliers of goods and services
3. Cost of treating water
4. Suitability of site for development which includes zoning, utilities and space.

To construct this model, matrices must be constructed whose cells reflect the status of each site. Let the matrix T contain for each cell the cost-distance from each parcel to the nearest terminal. If the parcel is suitable, enter the value, otherwise the cell has a value of 0. Construct a matrix S such that each cell contains the cost, price and transportation cost from the nearest supplier of goods and services. If the cost value is suitable, enter this value in the cell, otherwise enter 0. In a similar fashion construct W and X where each suitable cell in W contains the water costs (or 0 if unsuitable) and each cell in X contains the taxes on the property (or 0 if unsuited).

Combining these matrixes:

$$K = T + S + W + X$$

and $_j$ is the total cost associated with each cell j.

Let i represent a cell where the products of the activity which we are locating are consumed. For each i, there exist the following known quantities:

C_i the current cost per unit of transporting goods from point of sale

U_i the number of units consumed by i

Now for any potential site j, ($K_j > 0$) calculate W_{ij} which is the cost per unit of transporting goods and/or services from point of sale j to consumer i. The sale for j is given by:

$$S_j = \sum_{k=1}^{n} U_k$$

where

$W_i < C_i$ and n is the number of cases where $W_{ij} < C_i$.

The gross income is:

$$I_j = P \cdot S_j = P \sum_{k=1}^{n} U_k$$

and profit by

$$\emptyset_j = I_j - K_j = P \sum_{k=1}^{n} U_k - K_j$$

Finally calculate $\emptyset_j$ for all j where $K_j > 0$ and the site selected, S is given by:

$$S = j\exists \quad \emptyset_j \text{ is maximum}$$

G. Migration

People move in and out of the urban systems on the bases of opportunities available to them. They move within the urban system or ultimately out of the area on the basis of the ability of the local system to satisfy given needs. One measure of population movement is the net population change of the urban system. This change can be represented as:

$$\Delta P_T = NG_T + IN_T - OUT_T$$

where

ΔP_T is the total population change in a specific time period

NG_T is the natural change (births - deaths)

IN_T is in-migration

OUT_T is out-migration

Further,

$$NG_{Tk} = K_k P_{k(T-1)} \quad \text{or} \quad NG_T = \sum_{k=1}^{n} K_k P_{k(t-1)}$$

where the natural growth rate is a function of the total population segregated by socio-economic class.

$$IN_T = \alpha J_{k(T-1)} \quad \text{or} \quad IN_T = \sum_{k=1}^{n} \alpha_k J_{k(T-1)}$$

where in-migration is largely a function of the number of jobs available in the system in the previous time period for each socio-economic class.

$$OUT_{TK} = \sum_{k=1}^{n} (MV-MS)_k + \alpha U_{dk} + \beta U_{rk}$$

where

$$MV = \sum_{i=1}^{n} \alpha P_{ki} + \sum_{i=1}^{n} D_i R_i$$

and

MV is the number of people who migrate

MS are movers who stay within the local system

αP_{ki} is the percent of the population by socio-economic class

D_i are people who are displaced by the razing of residences

R_i is a random number of people who move

αU_{dk} is the percent of under employed

βU_{rk} is the percent of unemployed

MS can be further described as:

$$MS = EI_{max} - \sum_{i-1}^{n} EI_i > 0$$

where

EI_{max} is the maximum available environmental index available to residences

EI_i is the present environmental index for a particular parcel

or

$$EI = PI + R_R + cRQ + dMS + eS_c + gT_x$$

where

EI is the environmental index

PI is the water pollution index

R_R is the rent

cRQ is the residence quality

dMS are movers who stay within the local system

eS_c are the residential services

gTx is the tax rate or welfare level

H. School Assignments

The school model considers the needs of the various socio-economic classes and compares these needs to the service levels of the schools. When there is more service available than the local district needs, the model parallels the concept of excess capacity in the plant or factory. Conversely, when there is not enough capacity in the public system, the shortages are assigned to the private sector, i.e., private education.

The process of school assignments or establishing school districts can be done in terms of supply and demand. In this light, several objective functions are possible such as minimizing cost, maximizing service along with containment within political boundaries, neighborhood boundaries and so forth. The simplist way to model is based on school districts. Let D_s be the total student demand by socio-economic class on all school districts then:

$$D_s = \sum_{k=1}^{n} \alpha P_{ksd}$$

where

α is the number of students per population class

P_{ksd} is the population by class by school district

Since the objective is to let $D_s = S_s$ where S_s are the total services that meet the total demand, we must consider the effective supply of each school within a district and the demand on that school. In other words $D_L/K_L < 1$ when $K_L = n$ and D_L is the demand on school L and K_L is the effective supply or capacity of school L given the quality thresholds of cost and socio-economic class for n students. The definition of K_L becomes:

$$K_L = \sum_{i=1}^{n} C_{dsd} R_{ki}$$

where

C_{dsd} is the physical capacity of the school plant in a specific school district

R_{ki} is the index for the quality of the school

For a second school, say m, let $D_m/K_m < 1$ and $K_m = m$ in the same manner as before and then determine:

$$K_m = \sum_{i=1}^{m} C_{dsd} R_{ki} - D_L$$

This process is repeated for all schools in the system and each K is summed to determine S_s. If

$$\sum_{j=1}^{m} K_j < S_s$$

then excess capacity exists and of course if

$$\sum_{j=1}^{m} K_j > S_s$$

this surplus must go to the private sector or the public school system must be expanded. In terms of cost, $S_L = (P_{pLT}, K_L, T_L$ where for school L, the cost of the physical plant (P_{pLT}), the cost of maintaining the quality threshold in teachers, supplies, etc., and the transportation costs for bussing must be minimized. This is performed by arranging the students within school districts to minimize the total cost of all schools in the system. If additional constraints are placed upon this objective such as segregation ratio, the model simply requires new school districts which do not minimize the costs due to excess bussing but do comply with ratio definitions.

I. Transportation

Transportation represents movements of people and goods to jobs and shopping locations. There are four modes of transportation considered in the journey to work. Namely car, bus, rapid rail and walking. The cost is measured in terms of time, money and inconvience (ecological distance). All work travel is calculated at peak hours. The traveling to and from commercial and industrial sites for non-employment purposes are calculated at off-peak hours and only consider car or truck and walking as alternative modes.

Since we are interested in minimizing travel costs, we first must consider Tew or the transportation to jobs. Let

$$Tew = \sum_{p_1=1}^{n} \min E_{p_1 p_2}$$

where

$E_{p_1 p_2}$ is the ecological distance between points P_1 and p_2

also let

$$E_i = A_i T_i + M_i + C_{(t-1)_i}$$

where

i equals trips

A is the dollar value of time T

M is the money costs of travel

C is dynamically adjusted congestion factor for iterative trips, i.e., historical value

If T_s is the transportation to shopping and commercial centers, then let

$$T_s = \sum_{P_1=1}^{n} \min C_{P_1 P_2}$$

where

C is the money cost of travel between points P_1 and P_2. Let MT_i be the money cost of a specific trip such that

$$MT_i = \min (H + B + C + W)$$

where

H is highway

B is bus

C is rapid rail

W is walk

Finally the total transportation cost T is

$$T = (Tew \sum_{i=1}^{n} E_i + T_s) \sum_{i=1}^{n} MT_i$$

where the cost to work is

$$T_W = Tew \sum_{i=1}^{n} E_i \sum_{i=1}^{n} MT_i$$

and the cost for shopping is

$$T_{shop} = T_s \sum_{i=1}^{n} MT_i$$

The travel for industrial or commercial for non-employment purposes is, of course, found in T_s with rapid rail eliminated as a possibility in MT_i.

J. Value of Structure and Capital Equipment

The fixed capital assets are assumed to decrease in value or depreciate each time period. Expenditures to repair or modernize the asset will negate the depreciation at the time period the improvement was made and the depreciation will begin again for the asset's new value.

The value of an asset at time t + 1 is:

$$V_{t+1} = NV_t - DL_{t+1}\, NC + M_{t+1}$$

subject to

$$V_{t+1} \leq NC$$

where

NC is the original cost

NV_t is the present value in a given time period

DL_{t+1} is the depreciation during a given time period in percent of original cost

M_{t+1} is the maintenance expenditure made during a time period.

Depreciation or land use can be determined as:

$$DL_{p\ell} = \sum_{\ell=1}^{n} [d_{ie}MS_{dmsp} + d_{2e\ell p} + d_{3e}FL_{KP} + d_{4e}FR_{dup} + d_{5e}U_p]$$

where

$DL_{p\ell}$	is the total depreciation for each land use on each parcel in the system
$d_{ie}MS_{dmsp}$	is the municipal service quality index per parcel by land use
$d_{2e\ell p}$	is the coefficient of time by land use type
$d_{3e}FL_{KP}$	is the depreciation of land use due to floods
$d_{4e}FR_{dup}$	is the depreciation due to inadequate fire protection
$d_{5e}U_p$	is the depreciation due to inadequate utility capacity

K. Water

The water sector is designed to replicate resources of surface water, rivers, lakes and run-off. By relating the supply of water of a given quality to the demand for that quality of water, the need for water supplies is adjusted by importing water and upgrading the quality of available water.

The demand (D_d) is given by:

$$D_d = \sum_{p=1}^{n} L_p(WC_p) + \sum_{p=1}^{n} (P_{KP})(WC_{\ell KR}) + L_{SW}(We)$$

where

D_d	is the demand for water in a given water district
L_p	is land use on a parcel
WC	is water consumption
P_{KP}	is the population of a given class or a parcel

L_{SW} is the imported water

We is the water quality

The supply (S_d) is given as:

$$S_d = LV_t(TR_d)\left[\frac{A_p}{D_p}\right]$$

where

S_d is the supply of water in a district

LV_t is the land value of a particular parcel

TR_d is the transportation value to a given terminal

A_p is available water on a parcel

D_p is the total demand on a parcel

The total water supply is then found as:

$$T_W = \sum_{d=1}^{m} S_d + \sum_{p=1}^{m} W_{sp} + S_o$$

where

W_{sp} is surface water

S_o outside purchase water

If the demand is greater than the supply, there is a tendency to buy outside water such that:

$$\text{local } RW_p = A(D_p/S_p)$$

where

A = cost of bringing the present quality of water up to the desired quality

If the quality of water is less than some standard, or if

$$RW_p > 1 \quad \text{then}$$

$$S_o - D_p - S_p$$

VII. POST SCRIPT

As stated in the Introduction, further clarification of the terms, game procedures and examples of computer outputs will be found in the Computer Operator's, Director's and Player's manuals associated with each game. The primary purpose of this manual is to give the serious reader a detailed understanding of the mathematical models associated with the games and not the computer programming techniques utilized to make the models operational.

It is hoped that the user might utilize some of the models in his own programs or modify them for his own needs. Each major model can be considered and analyzed apart from the actual CITY game. The programming becomes complex only when all the models are integrated within a single game, such as CITY IV. It is then necessary to account for the inter-relations between models as well as the model solutions themselves.

To further help the reader, a Glossary of Terms and a List of Symbols is included in this Chapter. Only those terms which are unique to the Game or complicated in their definition are listed. Terms such as School Assignments or Transportation are defined in the text in their usual sense and have not been included in the Glossary.

Everywhere possible, the actual computer symbols are used. A few symbol changes have been made in this manual where a symbol in a main program might be the same as a symbol used in a sub-routine or function.

A. Glossary of Terms

1. Average Rate of Return - The expectation over the longrun for each investment made by an investor. Since some investment might not be profitable in the first few years and extremely profitable in the last years, a more representative yield on the investment would be the average return over the entire life of the investment.

2. Budget Allocations - In the Game context, the Government has a fixed amount of money at the beginning of each round. This money, acquired from debt and income, must be distributed to each governmental department in its entirety to avoid wastefulness. The proportionate distribution to each department becomes the budget for that round.

3. CITY I - A two module game consisting of the Economic and Government sectors played on an IBM 1131 computer. Manuals are available for purchase describing the details of this game at the National Technical Information Service (NTIS).

4. CITY II - This game is essentially CITY I with a partially operative Social Sector and operates on a Univac 1108, Exec. 8 computer from a terminal. There are presently no manuals available for sale for this game but arrangements can be made through NTIS for acquiring a set.

5. CITY III - A three module game consisting of the Economic, Government and Social sectors played on a Univac 1108, Exec. 8 computer. Manuals for this game are available for purchase from NTIS.

6. CITY IV - CITY IV is for the most part identical to CITY III and is written for an IBM 360/70 computer. Manuals for this game are available for purchase from NTIS.

7. City Model - The City models are operational simulation games in which the participants or players make decisions that affect a hypothetical metropolitan area built into the computer. CITY I, II, III, IV and the River Basin are different versions of the City Model.

8. Commercial Expenditures - These are the sum of the utility, tax, maintenance, water, salary, transportation, and goods and service costs.

9. Commercial Income - The income for any commercial establishment is the product of the number of units sold and the price per unit.

10. Computer Model Equations - These are the generalized computer equations for supply and demand. The format of the general equations is followed throughout the models.

11. Decision-Making Bodies - In the gaming context, the decision makers are the players. When one or more players are assigned a role, such as the School Department, the players are then known as a decision-making body.

12. Departmental Ratios - Ratios that relate supply and demand to education, adult education, municipal services, welfare payments, utilities and transportation.

13. Director's Manual - A City Games manual written expressly for the Game Director. The manual contains the technical details for running a game including the computer commands and options open to the Director for various gaming plays. Each game has its own unique Director's manual.

14. Expected Value - A player investment criteria used to express the average value of the net rate of return on all investments made specific land parcel.

15. Fixed Resources - These resources include water, land, people, buildings, roads, networks (utilities, transportation, communication, etc.), capital equipment and the related maintenance items necessary to keep them in operation.

16. Government Decision Objective - To maximize the total benefits to the system through the allocation of funds among the various departments.

17. Grid Matrix - In all models, a grid or checker-board arrangement is used. Each square or grid is used to identify and locate the numerous interrelationships, specific parcels of land and roads. The matrix then becomes the entire checker-board or the sum of all the individual grids.

18. Housing Income - The income for a given residential property is the product of the amount of space used and the rent per space unit for the property.

19. Industrial Expenditures - These are the sum of the utility, tax, maintenance, water, salary, transportation, goods and services, and terminal usage costs and the expenses connected with pollution control.

20. Industrial Property Income - The income for a given industrial property is the product of the price per unit (industry specific) and the number of units produced on the properties.

21. Industrial Value Ratio - The value ratio is determined by the amount of depreciation that occurred during a given round limited by the level at which maintenance is set.

22. Investment Capital - In the game, investment capital or money that a player has to invest. The rational is that the net amount of investment capital available after all costs of borrowing are taken into consideration must be less than or equal to the estimated rate of return in dollars.

23. Investment Series - In the game, investment series refers to all of the investments made by a player in a given round. For example, if in round one a player invested $25,000 in various investments, this would be known as his investment series for round one.

24. Liquid Resources - Those items which can easily be changed in the short-run such as taxes, wages, sales, social attitudes and the like.

25. Matching - This is the process of locating workers by skill and educational level to fill the demand from the job market. The workers who are located must then be interested in the job offer because it will maximize their income and/or achieve their satisfying objectives.

26. Migration - People move in and out of the urban system on the bases of opportunities available to them. The game accounts for this in and out migration as well as the natural birth-death growth to consider the net population changes of the Urban System.

27. Modules - The computer program considers each major game function separately. For example, the Economic Sector is a module in terms of the computer programming. CITY III is a three module game consisting of the Economic module, the Government module and the Social module. These modules are referred to in the gaming instructions as Sectors. The distinction is made

so that if one refers to a sector, the game play and the players are referred to the Players' manual. The module terminology has meaning to the computer operator and the Game Director.

28. Money Stock - This term refers to the total money supply in the system at any given round of play.

29. Objective Function - This is the mathematical expression of the goals or standards set by each sector and the sub-division within a sector, such as the School Department's goals, for each game play.

30. Operator's Manual - A City Games manual written expressly for the computer operator. The manual contains directions, explanations and exhibits to guide the operator through a game play. Each game has it's own unique Operator's Manual.

31. Partial Resource Allocation - In a game play, it is not realistic to assume that every resource available would be changed. A resource committed in a prior game play may still be the best committment for the player. The player would only invest part of his remaining resources in the current round. The concept of not investing or altering all resources during a particular round is known as a partial resource allocation.

32. Player's Manual - A City Games manual written expressly for the player. The manual contains directions, explanations and exhibits to guide the player through a game play. Each game has its own unique Players' Manual.

33. Private Capital - Private capital or money is invested in fixed assets through three general categories: residential housing, commercial activities and industrial plant and equipment.

34. Private Liquid Capital - Liquid Resources were defined as those resources which could easily be changed such as taxes, wages, sales and the like. Private capital is non-government money invested by the private citizens and therefore Private Liquid Capital refers to the liquid capital of private investors.

35. Property Expenditures - The expenditures for a property are found by summing for each property the utilities, taxes, maintenance and water costs.

36. Public and Private Influence - This refers to the so called "power" or "political power" exerted by individuals or groups to gain an objective. The game considers voting, economic suasion, and influence peddling in the form of bribes and boycotts as the tools for a power base.

37. Rate of Return - The amount of money earned from an investment divided by the investment results in a percentage figure or rate of return. If a principal sum is invested and the net return from the investment is 10%, then the rate of return on the investment is 10% if a single payment or the rate can be in terms of compound interest over time.

38. River Basin Model - This game is CITY IV with the water module attached. It is the largest of the present series of games consisting of four modules.

39. Semi-Liquid Capital - Semi-liquid capital is another term meaning private liquid capital. The term arises from the definition of partial resource allocation and supplies that only some of the private liquid capital will be reinvested during a particular game round or play.

40. Structure Value - The structure value for a given property is the original cost of a level of the structure (type specified) times the value ratio times the number of levels on the parcel.

41. Time Allocation - The ability to allocate the leisure time of small aggregates of workers in the Social Sector. This time allotment produces complex interactive effects throughout the entire model and can become a significant source of worker "power."

42. Total Resources - The total resources of a particular system can be viewed as the sum of all fixed and liquid resources in the system.

43. Water Module - This module is added to the CITY IV game and considers the water distribution problems within the entire river basin rather than just the metropolitan area of the particular game in play.

B. List of Symbols

Symbol	Page(s)
A	63,66
aB	8
$(aB-dC)_i$	8
A_i	53
A_p	66
B	37,51,54,63
B_c	52
BG	21
B_i	53,54
$\hat{B}_i$	5,11
$\hat{B}_{ij}$	12
B_j	26
$BLDG_p$	19
$\hat{B}_p$	13,14
bPk	32
B_{ps}	45
BQI_f	55
BS	21
BTC	21
$B\bar{T}C$	22
BV	54
C	40,41,63
C_{AE}	36
C_{dsd}	61
CG	4
CH_i	53
C_i	5,9,57
C_{ij}	53
C_j	26
CP	4
C_p	13,14,16,43
C_{pg}	45
$C_{P_1P_2}$	63
cRQ	60
CU	21
CU_p	21
D	3,52
D_{AE}	29
DC	6,46
dC	8
D_d	27,65
dh_{ij}	3

List of Symbols (Continued)

Symbol	Page(s)
D_i	59
d_i	13
$d_{ie}{}^{MS}{}_{dmsp}$	65
dI_i	8
dI_{it}	10
$dI_{i(t-1)}$	10
dK	11
dK_i	11
dK_{ij}	12
D_L	61
$DL_{p\ell}$	65
DL_{t+1}	64
dm_i	3
D_{ms}	29
dMS	60
D_N	17
D_O	17
D_p	42,66
$D_{p'}$	42
D_s	60
D_{sc}	28
D_T	17
D_{TR}	30
D_{UT}	30
DZ	30
$d_{2e\ell p}$	65
$d_{3e}{}^{FL}{}_{KP}$	65
$d_{4e}{}^{FR}{}_{dup}$	65
$d_{5e}{}^{U}{}_{p}$	65
E	55
$E^{\bullet}$	46
EE	23
EE_{max}	24
EF_{cp}	29
EH	24
EI	55,60
$\bar{E}\bar{I}$	56
EI_{max}	59
EI_f	56
EI_i	52,56,60
E_j	51
$E_{k\,i}$	46
EL_c	35
$E\bar{N}$	11
$E\bar{N}_{max}$	11

List of Symbols (Continued)

Symbol	Page(s)
$\hat{E\bar{N}}$	12
$\hat{E\bar{N}}_{max}$	12
$E\bar{N}_j$	13
E_p	44
$E_{p_1p_2}$	62
ER	24
eSc	60
FAE_i	35
FR	3
FR_p	3
g_i	41
GS_p	20
gTx	60
G_y	52
H	63
i	3,4,9,40,57,63
I_{AE}	29
IG_ℓ	31
I_j	58
I_{ms}	29
IN	43
IND.	35
IN_T	58
I_{sc}	28
I_{TR}	30
I_{UT}	30
I_{WL}	30
j	3
J_{jk}	47
J_k	47,59
$J_{k(T-1)}$	59
JOB_{pt_c}	35
K	49,57
K_c	16
K_i	9
K_{ij}	12
K_j	57
K_k	58
K_L	61
$K\ell_i$	45

List of Symbols (Continued)

Symbol	Page(s)
L	21,62
LC	52
L_c	53
LC_i	52
Lg	54
Lg_{it}	54
L_I	53
L_i	31,45,46
L_{ij}	51
LND_p	18
L_p	18,19,29,30,65
LR	3
L_R	52
LRC	31
LR_i	3
L_{SW}	66
LU_i	45,54
LU_p	29,30
LV	54
LV_t	66
M	63
m	12
MG_j	26,31
m_j	13,14
M_{PG}	17
M_{PS}	17
MS	8,9,59
MS_i	53
MS_L	52
MS_p	30
MT_i	63
M_{t+1}	64
MT_p	17
MU	30
MV	59
n	8,11,12,46
$\bar{N}$	9
N_{max}	9
NBCPR	24
NC	64
N_c	34
NG_k	47
NG_T	58
NG_{Tk}	58

List of Symbols (Continued)

Symbol	Page(s)
N_i	9
N_{ij}	54
$\bar{N}_j$	14
NP_k	47
NP_m	24
NT_p	21
NV_t	64
O_h	19
OUT_T	58
OUT_{TK}	59
P	4,39,46
p	4,9,31
$\bar{P}$	46
P_c	29,34
PG	17,21,45
P_{gi}	39
PH	29
Ph_p	29
PI	60
P_i	13,31,46
p_i	40
P_{ik}	54
P_{jk}	48
P_{jkp}	48
P_k	40 ,49
P_{KP}	65
P_{ksd}	61
$P_{K(T-1)}$	59
PLA_i	37
PL_k	48
PM	29
P_m	23,24
P_{MLj}	35
Pm_p	29
P_{nk}	48
Po	46
POP_v	40
P_p	20
P_{pi}	39
P_{pLT}	62
PR	4,40
PR_c	36
PR_i	5
PRO	5

List of Symbols (Continued)

Symbol	Page(s)
P_{RT_j}	37
PS	17,21,45
P_s	47
PS_i	31
P_{sk}	48
PTN_p	24
P_T	58
PW_i	31
P_{xk}	48
$\bar{Q}\bar{I}$	56
QI_f	56
QI_i	56
R	47,55
$\bar{R}$	56
$\tilde{r}$	37
RA	18
RB	18
R_d	28
r_d	28
R_f	55
R_i	56,59
r_i	9
$\tilde{r}_i$	38
R_{ki}	61
r^2_p	43
R_p	16
r_p	42
r_{PC}	36
rP_p	36
R_R	60
RT_p	22
r_w	34
RW_p	66
S	57,58
S_{AE}	29
SAL_c	34
SC	7
SC_p	29
S_d	27,66
S_{epk}	48
S_h	18
S_i	8,56
S_{ij}	3

List of Symbols (Continued)

Symbol	Page(s)
S_j	51,57
S_L	62
SL_p	20
S_{ms}	29
S_o	66
S'_p	42
S_p	16,42
SP_c	29
SPT_k	49
S_s	61
S_{sc}	28
S_{TR}	30
S_{UT}	30
$S\phi$	51
T	4,30,32,57,63
$\tilde{T}$	37
t+1	64
T_A	32
T_{AF}	34
T_{AP}	34
TD	3
$TD_{(BS/BG)}$	45
Te_i	49
$T_{epp'}$	49
Tew	62
T_I	33
T_i	38,51,53
$\bar{T}_i$	52,53
$\tilde{T}_i$	38
T_L	62
TM_p	24
T_N	32
T_p	29
T_{PA}	34
T_{pc}	16
T_R	34,55
TR	3
TR_d	66
TR_j	26
TR_R	20,22
TS	46
T_s	63
T_{shop}	64
T_T	32,55
Tu	32

List of Symbols (Continued)

Symbol	Page(s)
T_W	66
T_w	34,64
TX_1	26
TX_2	26
TX_3	26
TX_4	26
TX_p	17
T_Z	33
U_i	57
U_p	19
UP_p	23
US_p	20
UT_p	17,30
UU	30
V	11,43,64
V_i	6,11
V_{ij}	12
V_k	47
V_p	13,14,19
VR	23
VR_i	46
VR_p	19,29,30
W	7,57,63
WC	65
w_d	28
We	66
W_i	53
W_{ij}	57
w_n	7
W_{sp}	66
WTR_p	17
X	6,57
x_n	6
Y	6
Y_d	52
y_n	6
Z	7,49,55
Z_i	56
Z_k	51
$Z\ell_i$	45
z_n	7

List of Symbols (Continued)

Symbol	Page(s)
α	4,31,61
α_1	52
α_2	52
α_3	52
(α,β,γ)	31
αCP	40
αd_k	59
α_k	59
αP_{ki}	59
αU_{dk}	59
β	4,31
βCG	40
βU_{rk}	59
γ	4,31
γ_1	34,54
γ_2	35,54
γ_3	36
γ_4	36
γ_5	36
γAF_c	35
γAP_c	35
γR_c	36
ΔD_p	42
ΔEL_c	35
ΔP_o	46
ΔP_s	47
ΔPS_c	36
ΔP_T	58
Δr_p	42
ΔS_p	42
ΔVT_c	36
δ_i	40
δT	40
ε	4
ε_1	54
ε_2	54
ε_3	54
ε_4	54
εP	40
ζ	31
θ_1	6
θ_2	7

List of Symbols (Continued)

Symbol	Page(s)	Symbol	Page(s)
λ_k	40		
μ	9		
Φ	42		
ϕ_c	5		
$\bar{\phi}_c$	56		
ϕ_j	58		

Jan. 5, 1972

BIG CITY NEWS

JANUARY 5, 1972

no. 1

BIG CITY NEWS

A3

HOUSING MIX

housing units 300 200 100

single-family townhouse, row garden apts 275

BIG CITY NEWS

A4

AUCTION NOTICE

The Exchange announces its annual Outsiders' Auction of six parcels of land, as listed. Maps have been ...sted showing locations ...king prices, toge... maps of the ...r other ...nd in the

low b... rathe... last ... bid... the... ac... If the ... The ... remarkable recovery this ... year in investment and ... financial indicators, ... despite continued severe unemployment.

The outside interest rate averaged 5.1% over the past year, and our government bonds were selling well at 3.1%. The Chamber's annual canvas of of corporations in the area shows a 6.1% average return on conservative investments in the national economy and an excellent return, averaging 8.9% on growth stocks (sneculative investments) which vary widely from year to year.

Mr. Bigg oozed with juicy expectations for the coming year; he interprets the price index movement as the beginning of a major upswing.

He also reported a very favorable "trade balance" for the year. "We sold $4 billion worth of goods, chiefly heavy industrial (half) and light industrial, to the national economy. Our purchases from outside were only $2.3 billion, far less, and that was chiefly the federal corporation taxes. Our personal and business goods and services have been keeping up well with

Con't on pg.2col.1

FINANCIAL PICTURE BRIGHT last SAYS BUSINESS SPOKESMAN

"It's an ill wind that blows nobody good," says Jack Bigg, president of the Chamber of Commerce. He was referring to the

FIRST CENSUS RETURNS SHOWN CONTRASTS, CITY AND SUBURBS

The figures illustrate the first results of the new census, contrasting a picture of rapid but generally health growth in the suburban county, and stagnation or decline in the city.

Big City now has a population of 1,035,000. The Suburbs are about half that figure, 513,500. This represents substantial growth for the county, but no real growth for the city.

Unemployment, a chief topic of conversation and concern for some months is concentrated entirely in Big City. The city

con't on pg.3,col.3

A2 BIG CITY N...

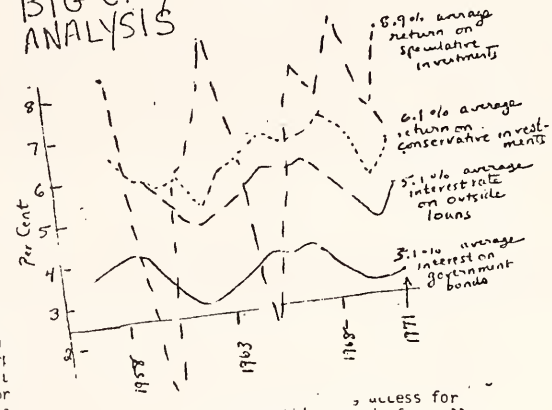
PERCENT

con't from pg.1,col. 2

our local demand, and consumers are satisfied with the service. We have had no boycotts or strikes during the past year."

Mr. Bigg expressed some concern that the national service sector- insurance finance/ R&D, real estate, etc. - were not expanding as rapidly here as in the nation.

BIG CITY STOCK MARKET ANALYSIS



con't from pg. 1,col.3

½ income people, in the poorer neighborhood, 2 out of five are unemployed. We are hurting"

This is the result of low-income families being simply left to occupy depreciated tracts such as _____ and _____. There is no low-income housing being built in the area. Certainly not in the suburbs. In the city we have overcrowding. The Councilman waved at the Quality INdex Map and the Zoning Map on his wall.

"In the county now we are providing jobs in industrial parks for county residents. Residents prefer jobs nearest to their homes, and since ...ig City is largely developed, all metropolitan growth favors full employment in the county."

"I do not for a moment agree with the Mayor on cutting welfare payments," said the Councilman. He said he did back the Mayor's proposals of last month, for the city to invest in land and utility service extension, to make some packages of undeveloped land available for industrial and job development in the city. He said he planned to discuss with area congressmen the pending bills which would subsidize low-income housing. Placing low-income housing in the county, near major industrial developments, would reduce unemployment.

CONSULT... FOR ZOO... mends c... facilit...

The STudy ... deficient ... tional lan... not be sur... our institu... public libr... hospital, a... rented and ... space. Other cities average a square mile of public institutional land for 100,000 people. By this standard, Big City should have 10 sq. miles, and the coulty anether 5 square miles. We have none.

With respect to park land, the metropolitan area is better supplied. We have nearly 6 square miles of parks of the 111 square/mile/to land area in the city, of 514 square miles in the county.

The Bunk Foundation, endowed by the late John Bunk, the founder of the Bunk (B) Corporation, has already

...ge holiday crowds from all ... access for ... parts of the city.

The library must be centrally located, despite high land costs. Among the alternatives discussed in the report are sites near the old rented library downtown, future slum clearance sites in the harbor area, and existing park land in one of the large and more central squares in Big City, for example 36-102. The new map of Total Planning and Zoning Land and Public Institutional Land is on display at city hall.

Even on park land, the minimum unit of development for a public institution costs $1 mil.4% of a square mile. This minimum park land provides recreation space for 4800 time units of recreation. Federal-state aid can be obtained for park development subject to approvals

BIG CITY COUNCILMAN

Councilman Coolew , fr... Big City, in an exclusive interview this morning, c... mented on the Mayor's ta... "As I see it, this whole ... blem is related to the f... mental land development ... blems facing the metrop... area."

He scratched figures ... desk pad..."Only 20% fo... county labor force is ... skilled or low class, f... of the city labor forc... This is where the unem... ment hits," said the c... man. "The jobs are ta... first by the high inc... ple, then by the midd... In Big City, one wage ... in five is unemployed ... this is all among the...

con't pg. 2, col.

BIBLIOGRAPHIC DATA SHEET	1. Report No.	2.	3. Recipient's Accession No. COM-74-10703-1

4. Title and Subtitle
CITY GAMES
MATHEMATICAL FOUNDATIONS

5. Report Date
September 1973

6.

7. Author(s)
Mr. John E. Moriarity, Editor

8. Performing Organization Rept. No.

9. Performing Organization Name and Address
National Bureau of Standards
Department of Commerce
Washington, D. C. 20234

10. Project/Task/Work Unit No.

11. Contract/Grant No.

12. Sponsoring Organization Name and Address
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

13. Type of Report & Period Covered

14.

15. Supplementary Notes This manual is to be used along with CITY I or CITY IV Games, Player's, Director's and Operator's manuals.

16. Abstracts The CITY models are operational simulation games in which the participants make economic, government and social decisions affecting a hypothetical metropolitan area. Through the use of a computer, the simulated urban system responds to the participant's decisions as any real city would. To simplify the description of the theory of the CITY GAMES modules, this manual is written for the four module game. The equations for the other games can easily be obtained by eliminating the descriptors that do not apply to the particular game of interest. The theoretical structure of all models is consistent and relevant only within the range from a central city area to a regional configuration. The description of the module components and equations are meant to show the scope of decisions including those by the user and by the programs and algorithms of the computer model.

17. Key Words and Document Analysis. 17a. Descriptors

* Computers
* Simulation
* Mathematics
* Equations

17b. Identifiers/Open-Ended Terms

* Mathematical foundations
* Computer Programs
* Computer simulation
Simulation module

17c. COSATI Field/Group

18. Availability Statement This manual is available only with the lease of the CITY I or CITY IV Games systems, COM-74-10701 and COM-74-10706.

19. Security Class (This Report) UNCLASSIFIED

20. Security Class (This Page) UNCLASSIFIED

21. No. of Pages 92

22. Price

FORM NTIS-35 (REV. 10-73) ENDORSED BY ANSI AND UNESCO. THIS FORM MAY BE REPRODUCED USCOMM-DC 8265-P74

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